Luttinger liquid with strong spin-orbital coupling and Zeeman splitting in quantum wires

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We study a one-dimensional interacting electron gas with the strong Rashba spin-orbit coupling and Zeeman splitting in a quantum well. A bosonization theory is developed for this system. The tunneling current may deviate from a simple power law which is that in an ordinary Luttinger liquid. The microscopic interacting coupling and the spin-orbital parameter may be measured by varying the external magnetic field in the tunneling experiment.

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The realization of the quasi-one-dimensional conductor in quantum wires, carbon nanotubes and DNA molecules provided possible realization of Luttinger liquid systems [1]. Quantum wires fabricated from narrow gap semiconductors, e.g., InAs, are highly interesting objects in the spintronics due to their rich spin transport properties and their application as spin transistors.

The Rashba effect [2], arising from the confining potential which is necessary to fabricate the quantum wires, dominates the behavior of the spin-orbit(SO) coupling in narrow gap semiconductors with heavy and light hole bands. Moroz and Barnes have shown that in a strong Rashba spin-orbital coupling, the dispersion of the electron in the quasi-one dimensional quantum well is drastically deformed [3]. The left and right Fermi velocities are not equal for a spin-s electron while the dispersions of the spin-up and down electrons have a mirror symmetry. Furthermore, a bosonization theory including such a strong Rashba spin-orbit coupling has been constructed[4]. It was found that, instead of the separated spin and charge excitations, two new branches of excitations with mixed spin and charge were found and the characteristic of the Luttinger liquid are modified by the SO coupling parameter [5].

To detect the SO coupling, an external magnetic field was applied [6]. Moreover, reliable techniques to measure the SO parameter need to be further developed. On the other hand, the Zeeman splitting due to the external magnetic field also modifies the transport behavior of the Luttinger liquid [7]. A further question raised here is what will happen if both the SO interaction and Zeeman splitting are considered simultaneously. We find that a bosonization technique combining the methods used in Moroz et al [4] and Kim et al [7] can be employed to solve the present problem. It is seen that there are four independent chiral excitations with different sound velocities. In an interacting electron system, this leads to different power laws for the left and right moving single particle densities of states (DOSs) as a function of the energy from the Fermi level. The tunneling current between a 3-dimensional metal(say, a STM tip) and the quantum wire may deviate from a simple power law and it may measure this exponent asymmetry in DOSs, and then the microscopic interacting coupling and the Rashba parameter.

Moroz and Barnes have computed the band structure of this model with an external magnetic field [3]. The non-interacting electron Hamiltonian under consideration reads

$$\begin{aligned} H_0 &= \frac{1}{2m_b}(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A})^2 + V(\mathbf{r}) + H_Z + H_{SO}, \quad (1) \\ H_Z &= -\frac{g}{2}\mu_B B\sigma_z \end{aligned}$$

where m_b is the electron band mass, g is the Lande g factor, and μ_B is the Bohr magneton. The vector potential $\mathbf{A} = -Bx\hat{y}$ which gives the magnetic field $B\hat{z} = \nabla \times \mathbf{A}$. The transverse confining potential is approximated by a parabola. The SO Hamiltonian is given by

$$H_{SO} = \frac{\alpha}{\hbar}[\vec{\sigma} \times (\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A})]_z, \quad (2)$$

where α takes $10^{-10} \sim 10^{-9}$ eVcm [2]. The strong Rashba effect appears if $l_\omega/l_\alpha \geq 1.4$ for $l_\alpha = \hbar^2/2m_b\alpha$ and $l_\omega = \sqrt{\hbar/m_b\omega}$ where ω is the frequency of the confining potential. In this case, Moroz et al found the linearized second quantized Hamiltonian for $B = 0$ can be written as [4]

$$\begin{aligned} H_0(B=0) &= \sum_k v_1 k(c^\dagger_{k,R,+}c_{k,R,+} - c^\dagger_{k,L,-}c_{k,L,-}) \\ &+ v_2 k(c^\dagger_{k,R,-}c_{k,R,-} - c^\dagger_{k,L,+}c_{k,L,+}), \quad (3) \end{aligned}$$

where the Fermi velocities $v_{1,2}$ are dependent on spin and chirality, which can be obtained by calculating the spectrum of the system. The difference between the velocities, $\delta v_F(\alpha,\omega) = v_1 - v_2$ monotonically increases as α is enhanced. The electron fields $c_{k,\gamma,s}$ for $\gamma = R, L$ and $s = \pm$ relate to $c_{k,\gamma,\delta}$ ($\delta = \downarrow, \uparrow$) by $c_{k,\gamma,\delta} = a_{\delta s}(\alpha,\omega,k)c_{k,\gamma,s}$, where $a_{\delta s}(\alpha,\omega,k)$ is a spin rotation matrix which can be obtained by numerically solving the single particle Schrödinger equation corresponding to eq.(1) [3]. It may be expanded as $a_{\delta s} = \delta_{\delta,s} + O(\delta v_F/v_F)$ with the average Fermi velocity $v_F = (v_1 + v_2)/2$.

For a weak magnetic field, its coupling to the orbital can be neglected if we consider the low-lying excitation. In the following, we only keep the Zeeman term with respect to the magnetic field. Define $k_F = m_b v_F$ and $\delta v_B = g\mu_B B/k_F$, the Zeeman term reads $H_Z = 2\delta v_B k_F \sum(\sigma_z)_{\delta\delta'} a^{*-1}_{\delta s} a^{-1}_{\delta' s'} c^\dagger_{k,s} c_{k,s'} \approx 2\delta v_B k_F \sum(\sigma_z)_{ss'} c^\dagger_{k,s} c_{k,s'} + O(\delta v_B \delta v_F/k_F^2)$. Thus, for $\delta v_F \sim \delta v_B \ll v_F$, the Zeeman term can be directly added to the Hamiltonian (3). The dispersion of the lowest subband now has a Zeeman splitting as sketched in Fig. 1. The linearized non-interacting electron Hamiltonian is given by

$$H_0 = \sum_{k,\gamma,s} v^s_\gamma k c^\dagger_{k,\gamma,s} c_{k,\gamma,s}, \tag{4}$$

where $v^s_\gamma = \pm(v_F + s\delta v_F/2) + s\delta v_B/2$ are four different sound velocities.

Bosonized Hamiltonian: Using the above known result, we are going to the bosonized formalism. We take the electron interaction to be the SU(2) invariant form $H_{int} = 2\pi U \int dy \psi^\dagger_\delta(y)\psi_\delta(y)\psi^\dagger_{\delta'}(y)\psi_{\delta'}(y)$ for the electron field. At low energies, the interaction can be divided into the forward, backwordard, umklapp and oscillating terms. For the semiconduct quantum well, it is enough to only keep the forward scattering in low energies [9]. In the bosonization theory, the electron operators $\psi_{\gamma,l}(x)$ are defined by $\psi_{\gamma,l}(x) \propto e^{-i(-1)^l 2\sqrt{\pi}\phi_{\gamma,l}(x)}$. Assuming q to be the low-lying excitation wave vector and $q \ll \delta v_F \sim \delta v_B \ll v_F$, the bosonized form of the whole Hamiltonian is easily arrived at:

$$\begin{aligned} H &= \frac{1}{2}\int dx \Big[v_\rho K_\rho (\partial_x\theta_\rho)^2 + \frac{v_\rho}{K_\rho}(\partial_x\phi_\rho)^2 \\ &+ v_\sigma K_\sigma (\partial_x\theta_\sigma)^2 + \frac{v_\sigma}{K_\sigma}(\partial_x\phi_\sigma)^2 \\ &+ \delta v_B(\partial_x\phi_\rho\partial_x\phi_\sigma + \partial_x\theta_\rho\partial_x\theta_\sigma) \\ &+ \delta v_F(\partial_x\phi_\rho\partial_x\theta_\sigma + \partial_x\phi_\sigma\partial_x\theta_\rho)\Big], \end{aligned} \tag{5}$$

where the subindices ρ, σ are the charge and s-spin degrees of freedom; $v_{\rho,\sigma} = [(v \pm U)^2 - U^2]^{1/2}$ and $K_{\rho,\sigma} = \sqrt{v/(v \pm 2U)}$. Formally, the bosonized Hamiltonian (5) is a direct generalization of the theory given in [4] or [7]. However, the physics included in such a generalization is richer.

Excitaions: After diagonalizing the Hamiltonian (5), the theory cannot be written as that of a standard harmonic fluid form of the Luttinger liquid [8]. We have four branches of the chiral excitations with the velocities u^l_γ, which are the absolute value of the solutions of the eigen equations. When $\delta v = 0$ or $\alpha = 0$, one gets back the cases discussed in Refs.[4] and [7]: There are two excitations whose velocities are $u^R_\pm = u^L_\pm = [v^2 + (\delta v_F/2)^2 \pm 2v\sqrt{U^2 + (\delta v_F/2)^2}]^{1/2}$ or $[v^2 + (\delta v_B/2)^2 \pm 2\sqrt{v^2(\delta v_B/2)^2 + v_1 v_2 U^2}]^{1/2}$, respectively. Using the sound velocities u^l_γ, one can solve the eigenvectors $\beta^{(\gamma,l)}_a$ which obey the sympletic orthogonal condition: $\sum_a (-1)^{a+1}\beta^{(\gamma,l)}_a \beta^{(\gamma',l')}_a = (-1)^{1+l}\delta_{\gamma,\gamma'}\delta_{l,l'}$. The old fields $\phi^{R,L}_{\rho,\sigma}(x)$ can be expressed in terms of the four new chiral fields $\Phi^l_\gamma(x)$ with velocities u^l_γ: $\phi^R_\rho = \beta^{(+,R)}_1\Phi^R_+ + \beta^{(+,R)}_2\Phi^L_+ + \beta^{(+,R)}_3\Phi^R_- + \beta^{(+,R)}_4\Phi^L_-$ and $\phi^L_\sigma = \beta^{(-,L)}_1\Phi^R_+ + \beta^{(-,L)}_2\Phi^L_+ + \beta^{(-,L)}_3\Phi^R_- + \beta^{(-,L)}_4\Phi^L_-$ and so on. In these new fields, the Hamiltonian (5) is diagonalized.

Single particle correlation functions and tunneling current: The electron correlation function is a function of both the SO coupling and the external magnetic field. We first calculate the single particle correlation function , the Fourier transform at $x = 0$ of which corresponds to the single particle DOS. Using the relation between the old and new fields, the $s(=\pm)$-dependent correlation functions are given by

$$\begin{aligned} G_{+,R}(x,t) &\sim x^{-(\beta^R_1)^2}_{+,R} x^{-(\beta^R_2)^2}_{+,L} x^{-(\beta^R_3)^2}_{-,R} x^{-(\beta^R_4)^2}_{-,L}, \\ G_{-,R}(x,t) &\sim x^{-(\delta\beta^R_1)^2}_{+,R}, x^{-(\delta\beta^R_2)^2}_{+,L} x^{-(\delta\beta^R_3)^2}_{-,R} x^{-(\delta\beta^R_4)^2}_{-,L}, \end{aligned} \tag{6}$$

where $\beta^R_a = \frac{1}{\sqrt{2}}(\beta^{(+R)}_a + \beta^{(-R)}_a)$ and $\delta\beta^R_a = \frac{1}{\sqrt{2}}(\beta^{(+R)}_a - \beta^{(-R)}_a)$. The spin($\delta$) dependent correlation functions are given by

$$\begin{aligned} G_{\uparrow,R} &= |a_{\uparrow,+}|^2 G_{+,R}(x,t) + |a_{\uparrow,-}|^2 G_{-,R}(x,t), \\ G_{\downarrow,R} &= |a_{\downarrow,+}|^2 G_{+,R}(x,t) + |a_{\downarrow,-}|^2 G_{-,R}(x,t). \end{aligned} \tag{7}$$

Furthermore, since the spin rotational invariance, one has $G_{\rho,R}(x,t) = \sum_\delta G_{\delta,R}(x,t) = \sum_s G_{s,R}(x,t)$. The right-moving single particle DOS, which is given by the Fourier transform of $G_{\rho,R}(x,t)$ at $x = 0$, reads

$$n_R(\omega) \sim A_R \omega^{\beta_R} \tag{8}$$

where $\beta_R = \min\{\sum_a (\delta\beta^R_a)^2 - 1, \sum_a (\beta^R_a)^2 - 1\}$. Similarly, one can calculate $n_L(\omega) \sim A_L\omega^{\beta_L}$ with $\beta_L = \min\{\sum_a(\delta\beta^L_a)^2 - 1, \sum_a(\beta^L_a)^2 - 1\}$. In Fig. 2, we show the exponents β_l as functions of δv_B and δv_F. $\beta_R \neq \beta_L$ for all non-vanishing δv_F and δv_B. This is a new feature which was not observed before. The chiral DOS is an experimental observable. For example, in a tunneling current measurement between a 3-dimensional metal (say, an STM tip) and a quantum wire, the tunneling current is given by Fermi's golden rule:

$$I \propto \int d\varepsilon [n^{(3)}_R(\varepsilon) n_R(\varepsilon - eV) - n^{(3)}_L(\varepsilon - eV) n_L(\varepsilon)], \tag{9}$$

where $n^{(3)}_l$ is the single particle DOS of the Fermi liquid. At zero energies, the differential conductance is given by

$$\frac{dI}{dV} \propto n_L(eV) + \cos(\beta_R\pi) n_R(eV). \tag{10}$$

For an ordinary Luttinger liquid, $\beta_R = \beta_L$ and $dI/dV \propto n(eV)$, which has been recognized by Matveev and Glazman[12]. Now, due to $\beta_R \neq \beta_L$ while they are pretty close, the differential conductance is not a simple power law. Such an asymmetry of the exponents in DOSs and the complex in the differential conductance are a unique manifestation of the co-ordination between the Zeeman splitting and the SO precession in a Luttinger liquid. Furthermore, this differential conductance and the exponents measurements also imply that the interacting coupling U and the SO parameter α can be determined through the measured values of β_l^2 as a function of the external magnetic field. This may provide a reliable way to determine the SO parameter.

Density-density correlations: The density-density correlation functions describe the density fluctuations of the system. The charge and s-spin density fluctuations are given by

$$\begin{aligned} &\langle 0|\rho_{\rho,\sigma}(x,0)\rho_{\rho,\sigma}(0,0)|0\rangle \\ &\sim -\frac{K'_{\rho,\sigma}}{2\pi^2 x^2} + \text{const}\cdot\frac{\cos(4\pi k_F x)}{x^{2K'_{\rho,\sigma}}}, \end{aligned} \tag{11}$$

where $K'_\rho = (\beta_1^{(+R)} + \beta_1^{(+L)})^2 + (\beta_3^{(+R)} + \beta_3^{(+L)})^2 = (\beta_2^{(+R)} + \beta_2^{(+L)})^2 + (\beta_4^{(+R)} + \beta_4^{(+L)})^2$ and $K'_\sigma = (\beta_1^{(-R)} + \beta_1^{(-L)})^2 + (\beta_3^{(-R)} + \beta_3^{(-L)})^2 = (\beta_2^{(-R)} + \beta_2^{(-L)})^2 + (\beta_4^{(-R)} + \beta_4^{(-L)})^2$.

For$S^y = \frac{1}{2}\psi_s^\dagger \sigma^y_{ss'}\psi_{s'}$, the y-component of the s-spin operator, the correlation function is given by

$$\begin{aligned} \langle 0|S^y(x,0)S^y(0,0)|0\rangle &\sim \cos(4\pi k_F x)x^{-K'_\sigma/2} \\ &+ \text{const}\cdot x^{-2-(\frac{1}{2\sqrt{K'_\sigma}}-\sqrt{K'_\sigma})^2} \end{aligned} \tag{12}$$

Transport in an infinitely long pure wire : The electron transport in the Luttinger liquid is affected by the SO coupling and the external magnetic field. From the continuity equation, the current operator is defined by $j(x,\tau) = -\frac{i}{\sqrt{\pi}}\partial_\tau(\phi_{\rho,R} + \phi_{\rho,L})$ where $\tau = it$. Here we only show the calculation of the conductance for a pure infinitely long quantum wire. It is also straightforward to calculate the conductance with impurity scatterings[13], although we shall not show it here. The conductivity is defined by

$$\sigma_\omega(x) = \frac{e^2}{\pi\omega}\int_0^{1/T} d\tau \langle 0|T_\tau \partial_\tau\phi_\rho(x,\tau)\partial_\tau\phi_\rho(0,0)|0\rangle e^{-i\omega\tau} \tag{13}$$

Calculating $\langle 0|T_\tau\phi_\rho(x,\tau)\phi_\rho(0,0)|0\rangle$, one has the conductance $G = 2K'_\rho\frac{e^2}{h}$.

Similar to the charge current, we can define the s-spin current $j_\sigma(x,\tau) = -\frac{i}{\sqrt{\pi}}\partial_\tau(\phi_{\sigma,R}+\phi_{\sigma,L})$. The same calculation gives rise to the s-spin conductance $G_\sigma = 2K'_\sigma\frac{e^2}{h}$. The δ spin current are given by $j_{\rm spin}(x,\tau) = (|a_{\uparrow+}|^2 - |a_{\downarrow+}|^2)j_\sigma(x,\tau)$. The δ spin conductance in the low energy limit is given by $G_{\rm spin} = 2(|a_{\uparrow+}|^2 - |a_{\downarrow+}|^2)K_{\rm spin}\frac{e^2}{h}$ and $K_{\rm spin} = K'_\sigma$. The consequences arising from the above discussion are (i) For $\alpha = B = 0$, $j_{\rm spin} = j_\sigma$ and $G_{\rm spin} = G_\sigma \neq 0$ because of the SU(2) invariance of the system. (ii) $j_{\rm spin}$ and $G_{\rm spin}$ portray the B-dependence of the system while j_σ and G_σ portray the dependence of the system on the SO coupling. For $\alpha \neq 0$ and $B = 0$, the spin conductivities $G_\uparrow = G_\downarrow$, i.e., $G_{\rm spin} = 0$ if $B = 0$. When $U = 0$, this agrees with previous work[10]. For $\alpha = 0$, $j_{\rm spin} = j_\sigma$. The SO effect is reflected by G_σ which increases as U goes up. This enhancement of the SO effect has been observed for a two-dimensional electron gas [11]. (iii) When $B > 0$ and $\alpha > 0$, $j_{\rm spin} < j_\sigma$. On the other hand, $G_{\rm spin}$ grows as the interaction becomes stronger because G_σ grows. (iv) For given α (B), the magneto-polarization is given by

$$\begin{aligned} P &= \frac{j_\uparrow - j_\downarrow}{j_\uparrow + j_\downarrow} = (|a_{\uparrow+}|^2 - |a_{\downarrow+}|^2)\frac{K'_\sigma}{K'_\rho} \\ &= C\cdot\frac{\delta v_B}{v_F}\cdot\frac{K'_\sigma}{K'_\rho}, \end{aligned} \tag{14}$$

where $a_{\delta s}$ is a function of α, ω, k_F and B; C is constant with order one. It is seen that the interaction can strongly enhance the polarization. The detailed numerical analysis will be given in further works.

In conclusion, we have built a low energy effective theory when the Zeeman splitting and strong Rashba SO coupling in a Luttinger liquid are turned on simultaneously. Different from that in the ordinary two component Luttinger liquid, four chiral excitations with different sound velocities were found. An asymmetry of the single particle DOS has been found, which implies a deviate from the single power law of the tunneling current between a 3-dimensional metal and the quantum wire. The variation of the differential conductance as a function of the external magnetic field also allowed for the determination of the interacting coupling and the SO parameter. The concrete numerical results will be given elsewhere.

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[1] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[2] E. I. Rashba, Fiz. Tverd. Tela (S.-Peterburg) **2**, 1224 (1960)[Sov. Phys. Solid State **2**, 1109 (1960)].
[3] A. V. Moroz and C. H. W. Barnes, Phys. Rev. B **61**, 2464 (2000).
[4] A. V. Moroz, K. V. Samokhin, and C. H. W. Barnes, Phys. Rev. Lett. **84**, 4164 (2000); Phys. Rev. B **62**, 16900 (2000).

[5] However, for carbon nanotubes, the spin-charge separation is not destroyed, see, A. De Martino and R. Egger,Europhys. Lett. **56**, 570 (2001). It is also different from the weak Rashba coupling case where the spin-charge separation is held, see, e.g., W. Häusler, Phys. Rev. B **63**, 121310 (2001) and M. Governale and U. Zülicke, Phys. Rev. B **66**, 073311 (2002).

[6] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997). G. Engels, J. Lange, T. Schäpers, and H. Lüth, Phys. Rev. B **55**, R1958 (1997).

[7] T. Kimura, K. Kuroki, and H. Aoki, Phys. Rev. B **53**, 9572 (1996).

[8] F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).

[9] S. T. Chui and P. A. Lee, Phys. Rev. Lett. **35**, 315 (1975). In their notation, we have $U_\perp = U_{para}$ here.

[10] See, e.g., L. W. Molenkamp, G. Schmidt and G. E. W. Bauer , Phys. Rev. B **64**, 1212029(R),(2001). G. Feve, W. D. Oliver, M. Aranzana, and Y. Yamamoto Phys. Rev. B **66**, 155328 (2002).

[11] Guan-Hong Chen and M. E. Raikh, Phys. Rev. B **60**, 4826 (1999).

[12] K. A. Matveev and L. I. Glazman, Phys. Rev. Lett. **70**, 990 (1993).

[13] S. T. Chui and J. W. Bray, Phys. Rev. B **16**, 1329 (1977); Phys. Rev. B **19**, 4020 (1979). C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).

Fig. 1 The dispersion with both strong Rashba SO coupling and weak Zeeman splitting.

Fig. 2 (a) β_l as a function of the normalized magnetic field $\delta v_B/2 = g\mu_B/2k_F$. (b) β_l as a function of the spin orbit coupling strength δv_F. The circle is for $\alpha = 0$ ($\beta_R = \beta_L$). The filled(empty) up-triangle is for $\beta_R(\beta_L)$ $\delta v_F/v_F = 0.2$(a), $\delta v_B/v_F = 1/3$ (b). The filled(empty) down-triangle is for $\beta_R(\beta_L)$ with $\delta v_F/v_F = 0.4$ (a), $\delta v_B/v_F = 2/3$ (b). The interaction strength $U = 0.25 v_F$.